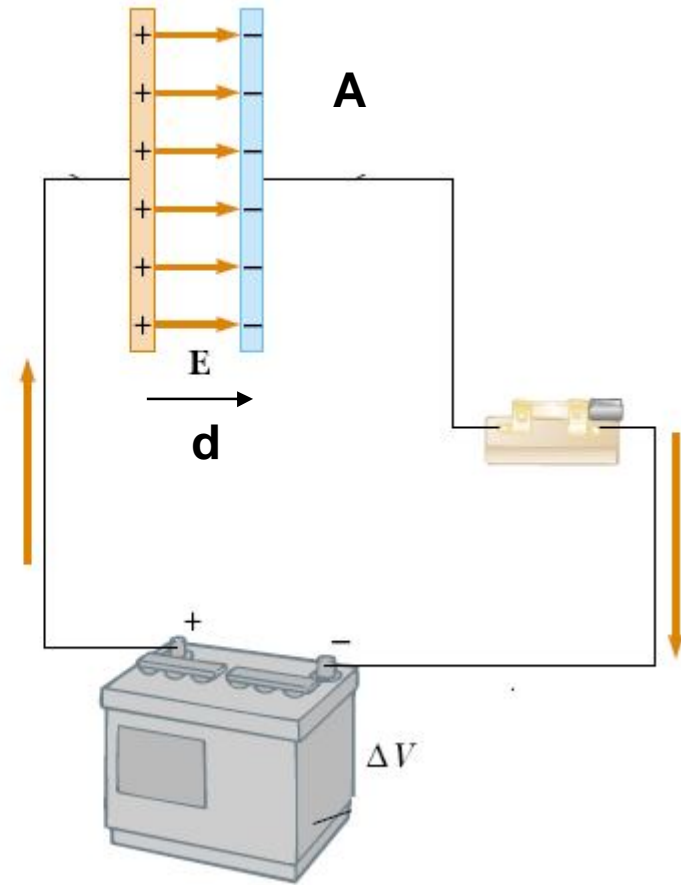
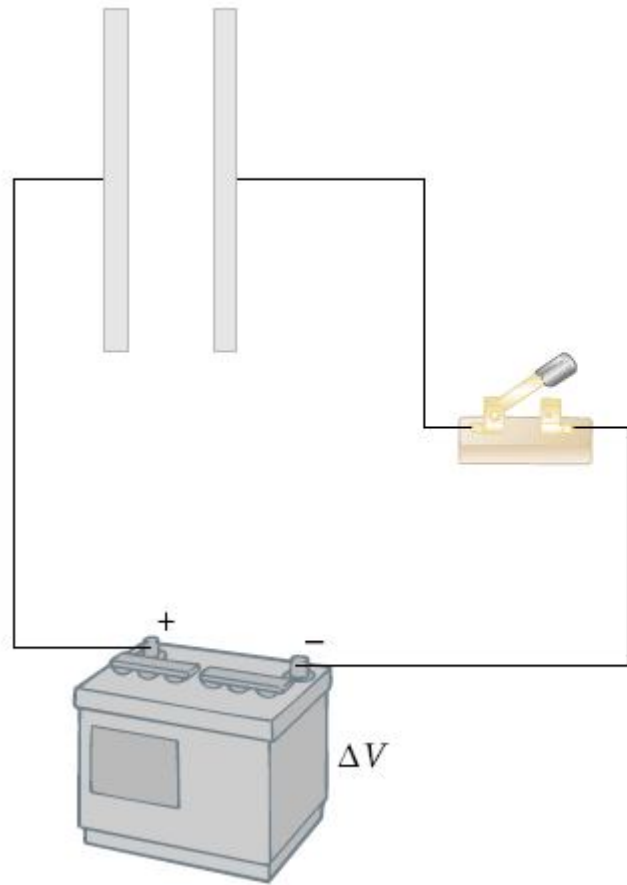


Dieléctricos

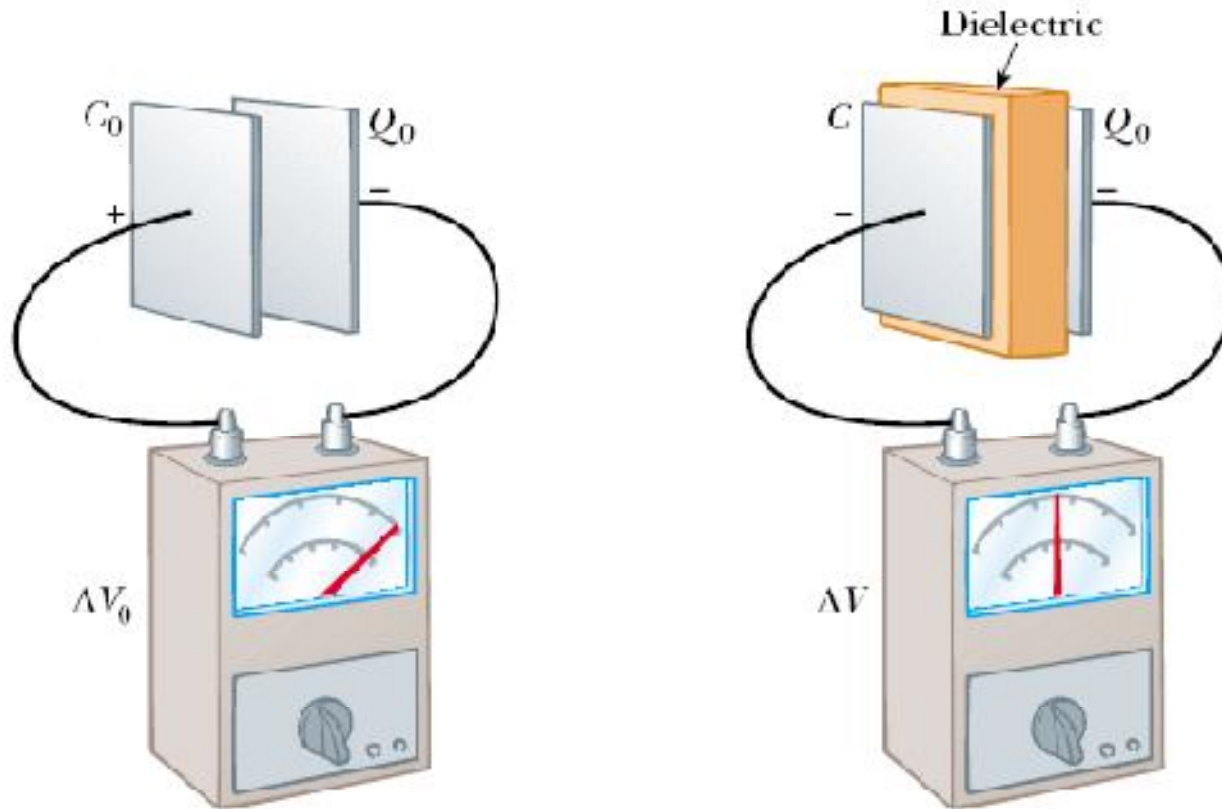
Bibliografía consultada

- **Sears- Zemasnky -Tomo II**
- **Física para Ciencia de la Ingeniería, Mckelvey**
- **Serway- Jewett --Tomo II**

$$C = \frac{q}{\Delta V} = \frac{\sigma \cdot A}{\Delta V} = \epsilon_0 \frac{A}{d}$$



Faraday comprobó

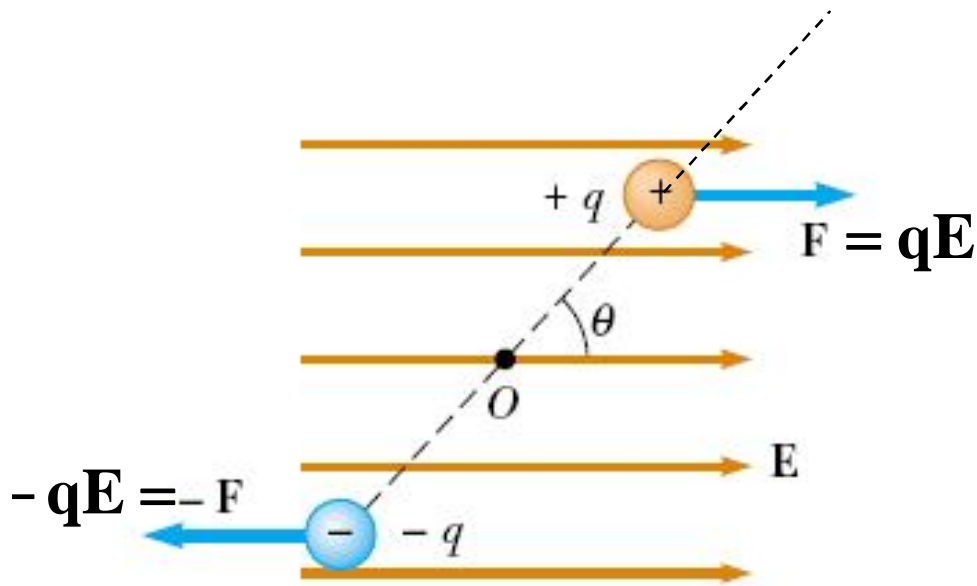
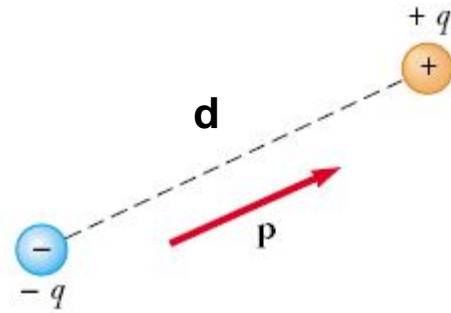


$$\Delta V = \frac{\Delta V_0}{\kappa}$$

$$C = \kappa C_0$$

DIELECTRICOS

INTERACCIÓN DIPOLO -CAMPO ELECTRICO



$$\sum \mathbf{F} = 0$$

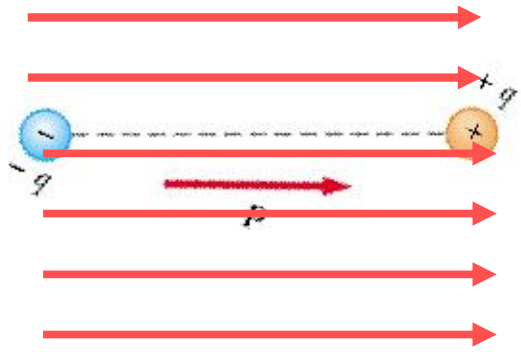
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau_+ = \frac{d}{2} F \sin\theta = \frac{d}{2} qE \sin\theta$$

$$\tau_- = \frac{d}{2} F \sin\theta = \frac{d}{2} qE \sin\theta$$

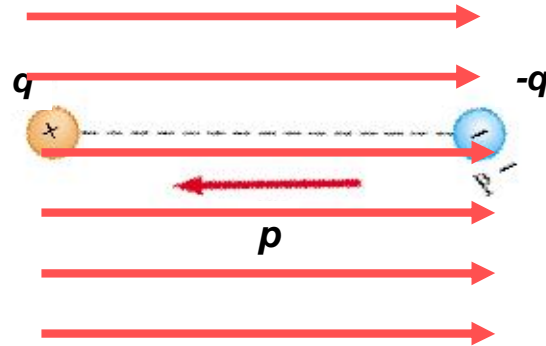
$$\tau = d qE \sin\theta = p E \sin\theta$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

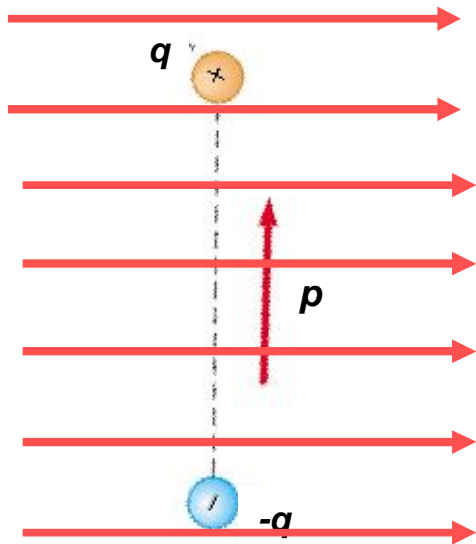


Eq. estable

$$\tau = 0$$



Eq. inestable



$$\tau \text{ es max. } = p E$$

El trabajo realizado para rotar el dipolo un ang. $d\theta$

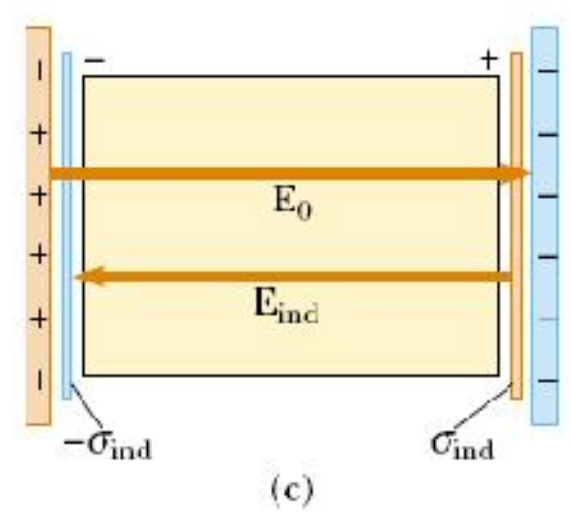
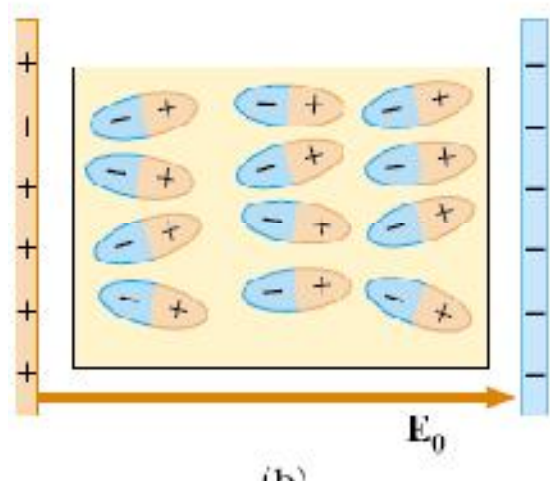
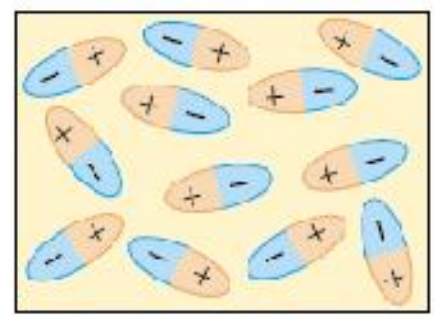
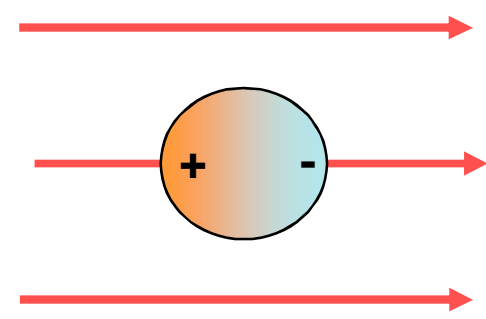
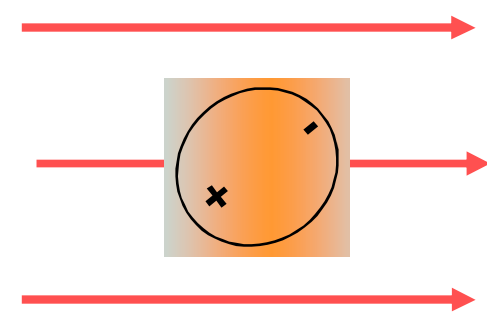
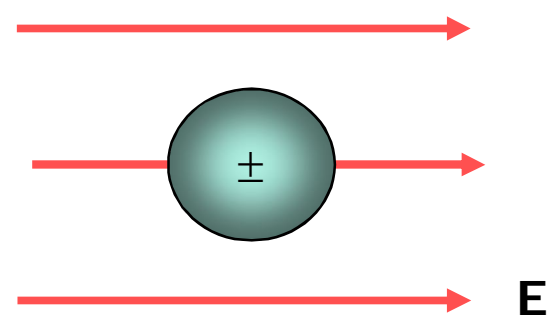
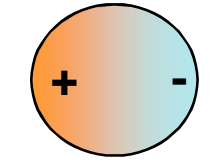
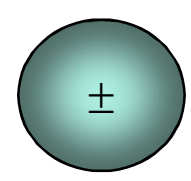
$$dW = \tau d\theta$$

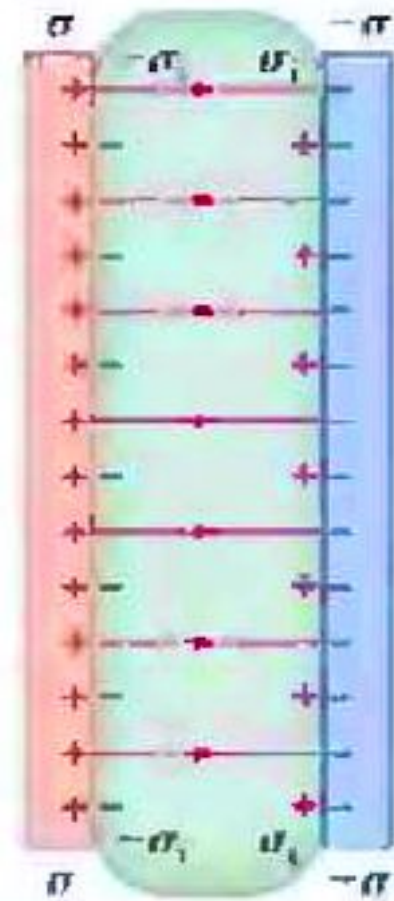
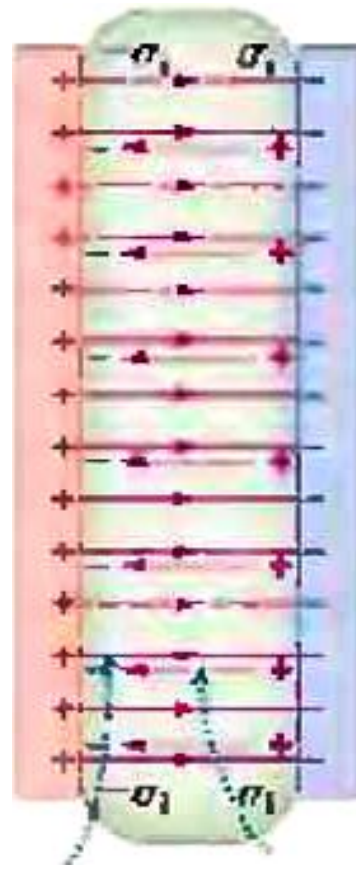
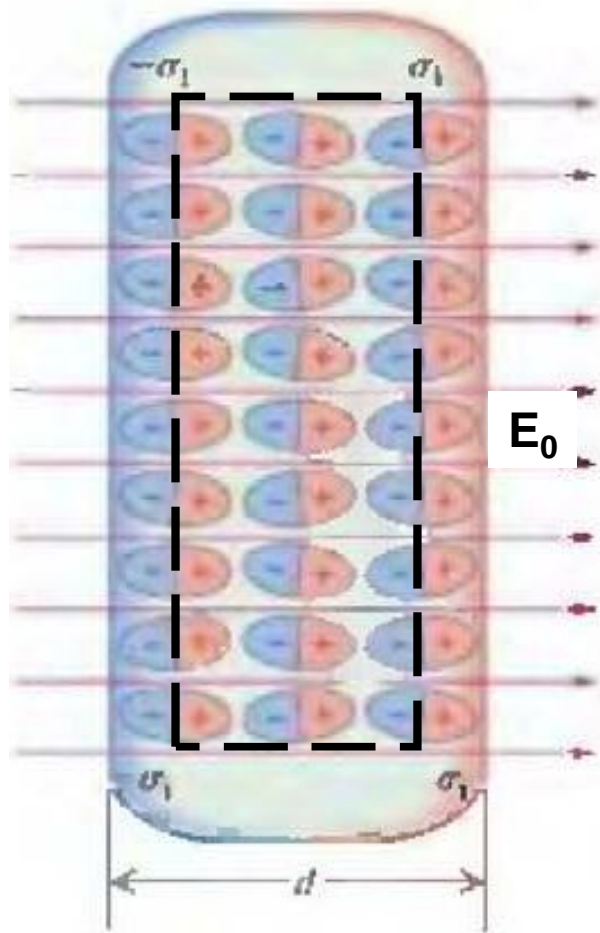
$$\Delta U = \int_{\theta_i}^{\theta_f} p E \sin\theta d\theta = p E (\cos\theta_i - \cos\theta_f)$$

$$\theta_f = 0 \quad \text{y} \quad \theta_i = 90$$

$$U = -\mathbf{p} \cdot \mathbf{E}$$

Las moléculas { Polares
No polares





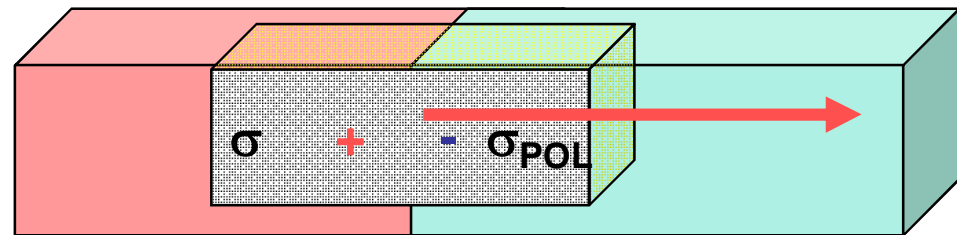
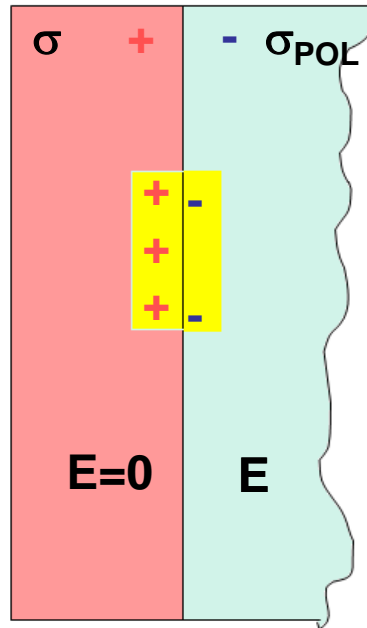
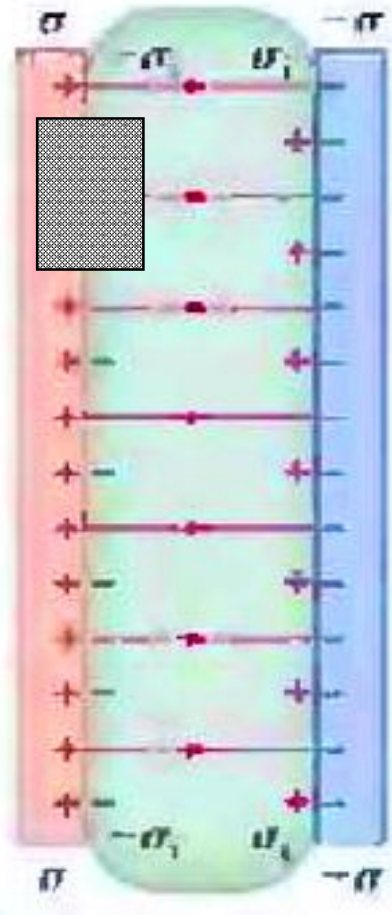
TEOREMA DE GAUSS

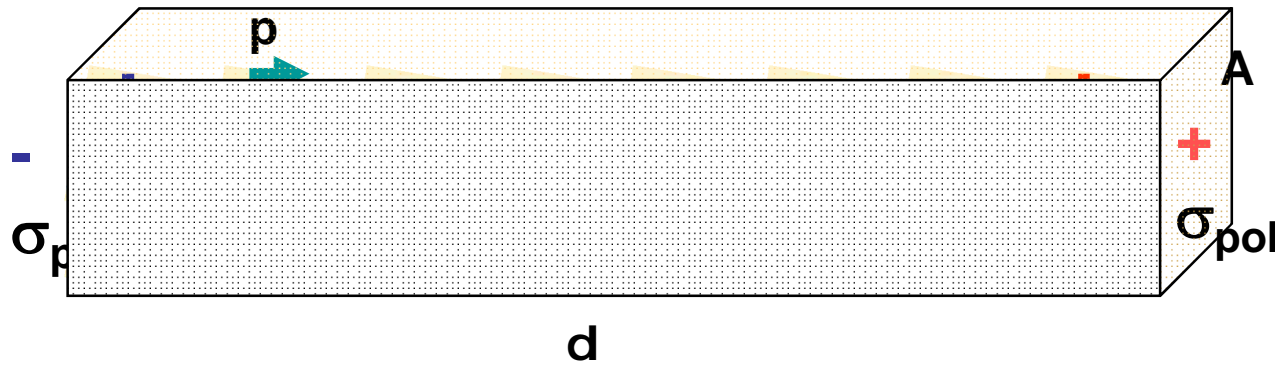
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = Q_L + Q_{pol}$$

$$Q_{enc} = \iint (\sigma_L + \sigma_{pol}) ds$$

↓
¿?



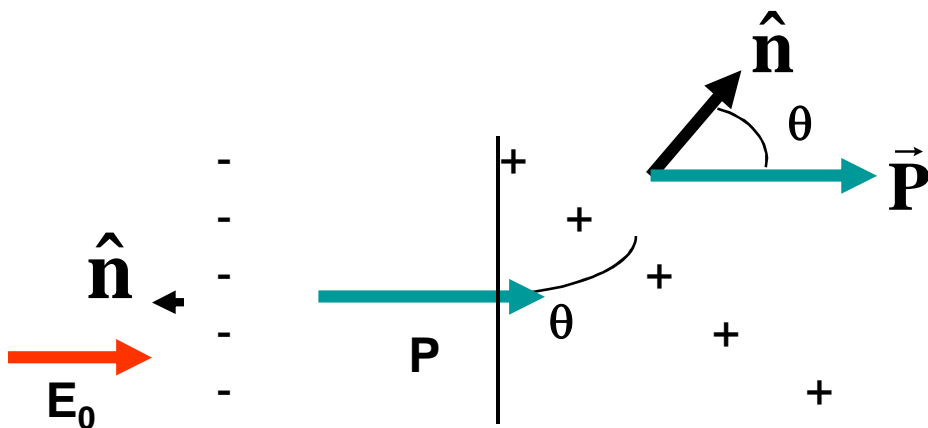


N moléculas con carga **q**

$$n = \frac{N}{\text{Vol}} = \frac{N}{A \cdot d}$$

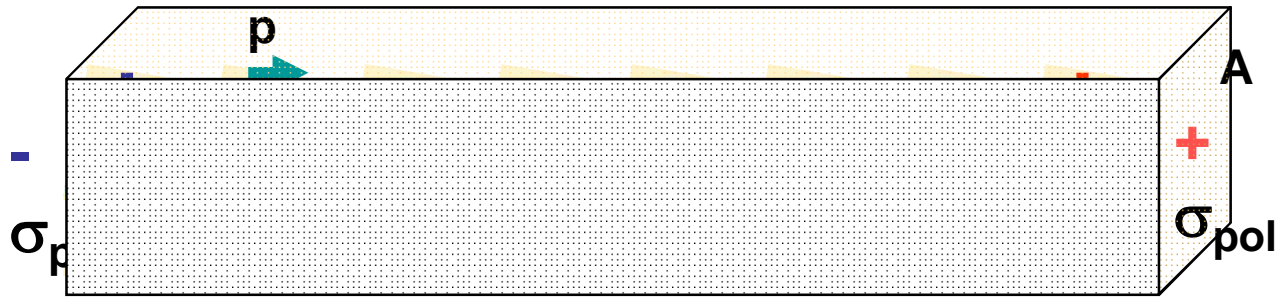
$$\Delta p = \sum_i p_i = q_{\text{pol}} d = \sigma_{\text{pol}} A d$$

Vector Polarización $\mathbf{P} = \lim_{\text{Vol} \rightarrow 0} \frac{\Delta p}{\text{Vol}} = \frac{\sigma_{\text{pol}} A d}{\text{Vol}} = \sigma_{\text{pol}}$



$$\sigma_{\text{pol}} = \vec{P} \cdot \hat{n}$$

Signo dado por la normal saliente

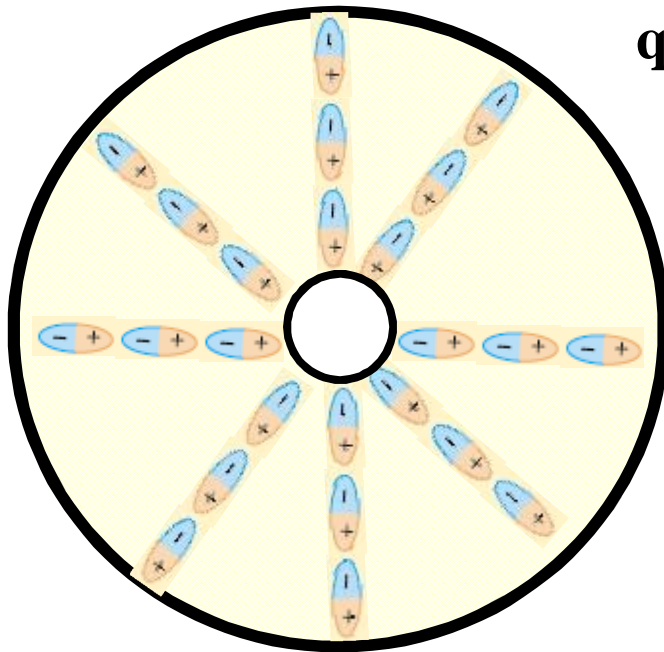


$$\sigma_{\text{pol}} = \vec{P} \cdot \hat{n}$$

$$dq_{\text{pol}} = \sigma_{\text{pol}} dA = \vec{P} \cdot \hat{n} dA = \vec{P} \cdot d\vec{A} \quad q_{\text{pol}} = \iint \vec{P} \cdot d\vec{A}$$

Si $P = \text{cte}$ $q_{\text{pol sup}} = \oiint \vec{P} \cdot d\vec{A} = 0$

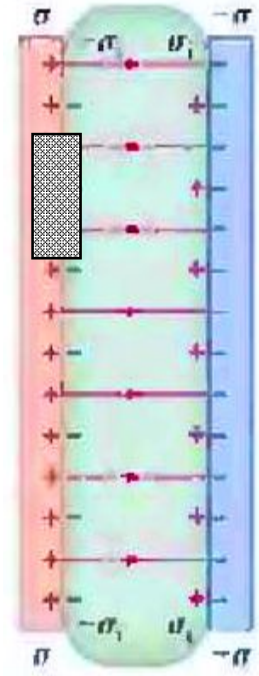
Si $P \neq \text{cte}$



$q_{\text{pol sup}} = \oiint \vec{P} \cdot d\vec{A} \neq 0$ Pero el dieléctrico estaba inicialmente descargado

$$q_{\text{pol vol}} = -q_{\text{pol sup}} = -\oiint \vec{P} \cdot d\vec{A}$$

$$q_{\text{pol vol}} = \iiint \rho_{\text{pol}} = -\oiint \vec{P} \cdot d\vec{A}$$



$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{q_L}{\epsilon_0} + \frac{q_{\text{Pol}}}{\epsilon_0} = \frac{q_L}{\epsilon_0} - \frac{1}{\epsilon_0} \oiint \vec{P} \cdot d\vec{A}$$

$$\oiint \epsilon_0 \vec{E} \cdot d\vec{A} = q_L - \oiint \vec{P} \cdot d\vec{A}$$

$$\oiint (\epsilon_0 \vec{E} + \vec{P}) \cdot d\vec{A} = q_L$$

$\underbrace{\hspace{10em}}_{\vec{D} \text{ Vector Desplazamiento}}$

$$\oiint \vec{D} \cdot d\vec{A} = q_L$$

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q_T}{\epsilon_0}$$

$$\oiint \vec{P} \cdot d\vec{A} = -q_{\text{pol}}$$

$$\oiint \vec{D} \cdot d\vec{A} = \iiint \rho_L d\text{vol} = \iiint \vec{\nabla} \cdot \vec{D} d\text{vol}$$

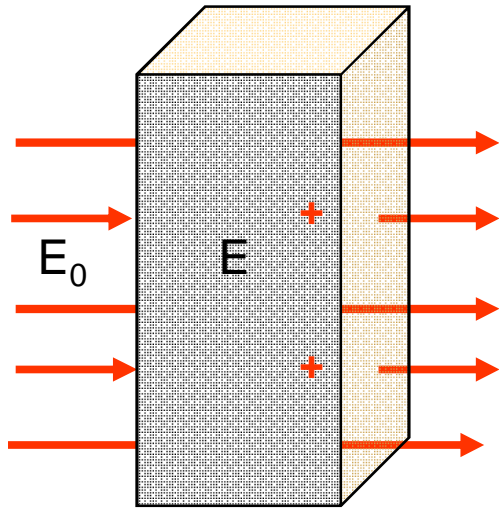
$$\oiint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \iiint \rho_{\text{total}} d\text{vol} = \iiint \vec{\nabla} \cdot \vec{E} d\text{vol}$$

$$\oiint \vec{P} \cdot d\vec{A} = -\iiint \rho_{\text{pol}} d\text{vol} = \iiint \vec{\nabla} \cdot \vec{P} d\text{vol}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_L$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{\text{Total}}}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{P} = -\rho_{\text{Pol}}$$



$$\mathbf{E} \leq \mathbf{E}_0$$

Susceptibilidad eléctrica
 $\chi > 0$

$$\vec{\mathbf{P}} = \chi \epsilon_0 \vec{\mathbf{E}}$$

Permitividad dieléctrica
del vacío

$$[\mathbf{P}] = \frac{\text{C}}{\text{m}^2}$$

$$[\mathbf{E}] = \frac{\text{N}}{\text{C}}$$

$$[\epsilon_0] = \frac{\text{C}^2}{\text{Nm}^2}$$

$$\vec{\mathbf{D}} = \epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}} \quad \vec{\mathbf{D}} = (\epsilon_0 + \chi \epsilon_0) \vec{\mathbf{E}} = \underbrace{(1 + \chi)}_{\epsilon_r} \epsilon_0 \vec{\mathbf{E}}$$

$$\vec{\mathbf{D}} = \epsilon_0 \epsilon_r \vec{\mathbf{E}} = \epsilon \vec{\mathbf{E}}$$

Medio	$\kappa = \epsilon_r$	$E_{\text{máximo}} \text{ (V/m)}$
Aire	1,00059	$3 \cdot 10^6$
Teflón	2,1	$60 \cdot 10^6$
Mylar	3,2	$7 \cdot 10^6$
Papel	3,7	$16 \cdot 10^6$

RESUMEN

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad \vec{D} = \epsilon_r \epsilon_0 \vec{E}$$

$$\vec{P} = \chi \epsilon_0 \vec{E}$$

$$\oiint \vec{D} \cdot d\vec{A} = q_L$$

$$\oint \vec{E} \cdot d\vec{L} = 0$$

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q_T}{\epsilon_0}$$

$$\oiint \vec{P} \cdot d\vec{A} = -q_{pol}$$

$$\oiint \vec{D} \cdot d\vec{A} = q_L$$

El flujo de D solo depende de las cargas libres
El flujo de D no depende del medio pero **D si!!!!!!**

$$\oint \vec{D} \cdot d\vec{L} = \oint (\epsilon_0 \vec{E} + \vec{P}) \cdot d\vec{L} = \oint \vec{P} \cdot d\vec{L}$$

$$\oint \vec{P} \cdot d\vec{L} = 0 \Rightarrow \oint \vec{D} \cdot d\vec{L} = 0 \quad \Rightarrow$$

D no depende del medio. Sólo de las cargas libres

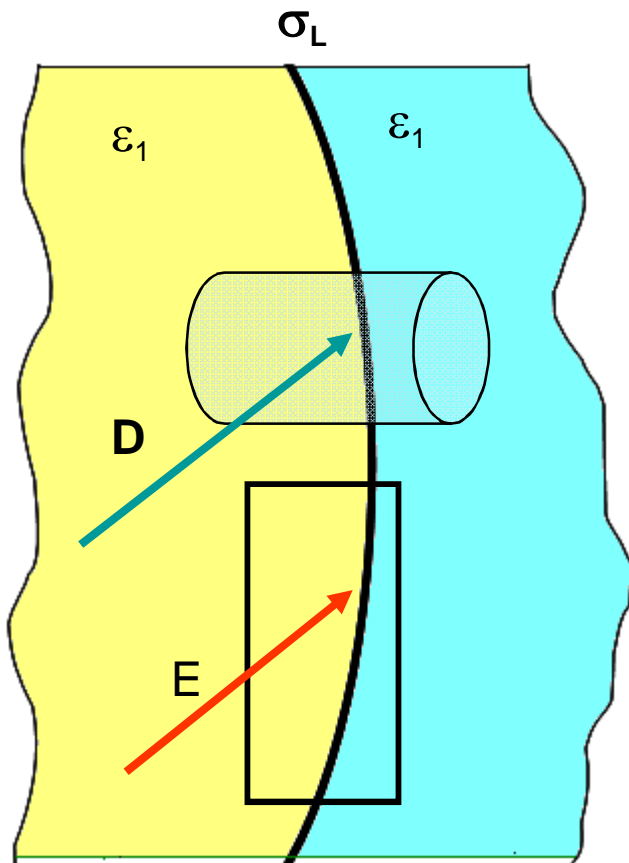
$$\vec{\nabla} \cdot \vec{D} = \rho_L$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{Total}}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{P} = -\rho_{Pol}$$

CONDICIONES DE CONTORNO



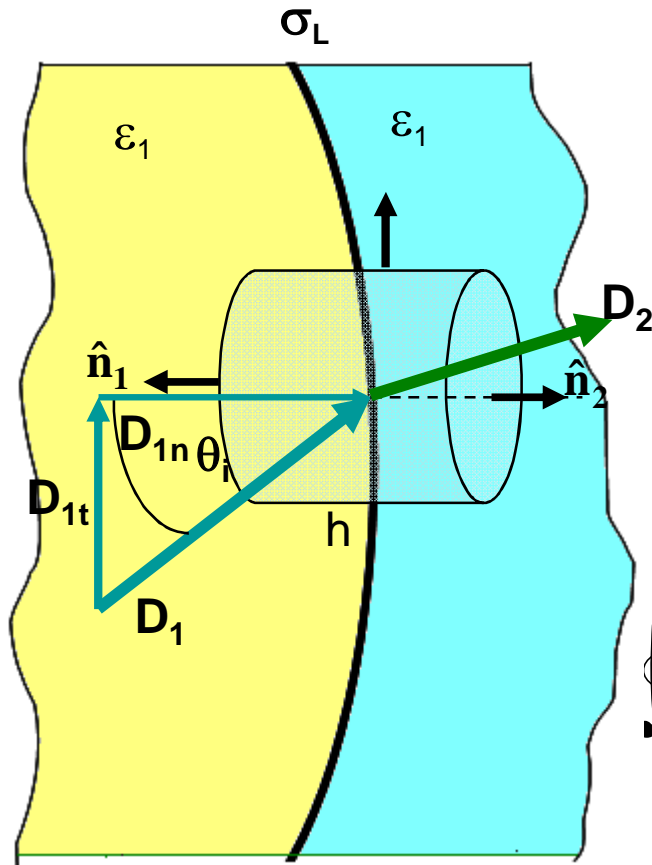
$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \chi \epsilon_0 \vec{E}$$

$$\oiint \vec{D} \cdot d\vec{A} = q_L$$

$$\oint \vec{E} \cdot d\vec{L} = 0$$

$$\oiint \vec{D} \cdot d\vec{A} = q_L$$



$$\oiint \vec{D} \cdot d\vec{A} = \iint_{\text{base}} \vec{D} \cdot d\vec{A} + \iint_{\text{tapa}} \vec{D} \cdot d\vec{A} + \iint_{\text{lateral}} \vec{D} \cdot d\vec{A}$$

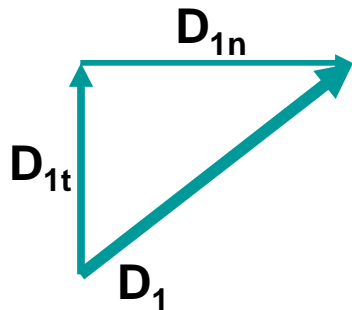
$$h \rightarrow 0$$

$$\oiint \vec{D} \cdot d\vec{A} = \iint_{\text{base}} \vec{D} \cdot \hat{n}_1 dA + \iint_{\text{tapa}} \vec{D} \cdot \hat{n}_2 dA$$

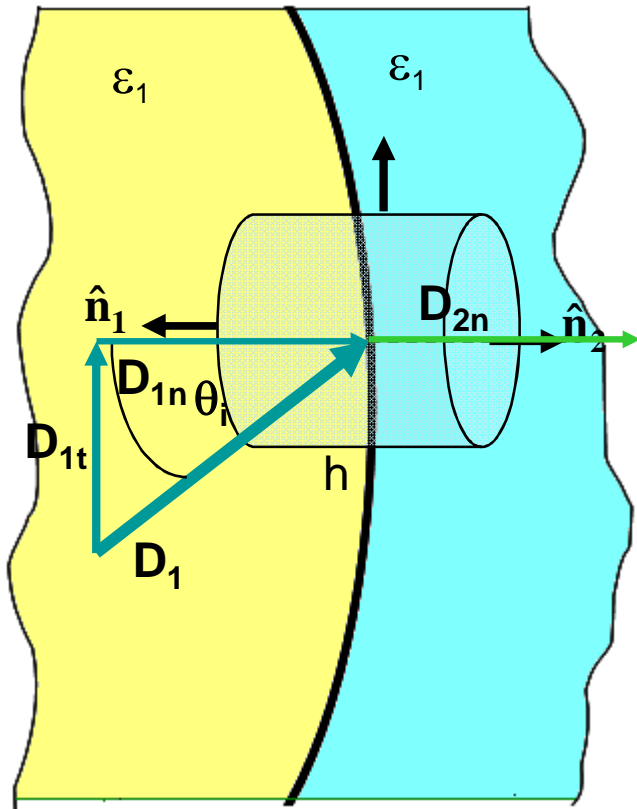
$$\oiint \vec{D} \cdot d\vec{A} = - \iint_{\text{base}} D_{1n} dA + \iint_{\text{tapa}} D_{2n} dA = \iint \sigma_L dA$$

$$D_{2n} - D_{1n} = \sigma_L$$

$$\epsilon_2 E_{2n} - \epsilon_1 E_{1n} = \sigma_L$$



$$\frac{\epsilon_{r2}}{\chi_2} P_{2n} - \frac{\epsilon_{r1}}{\chi_1} P_{1n} = \sigma_L$$



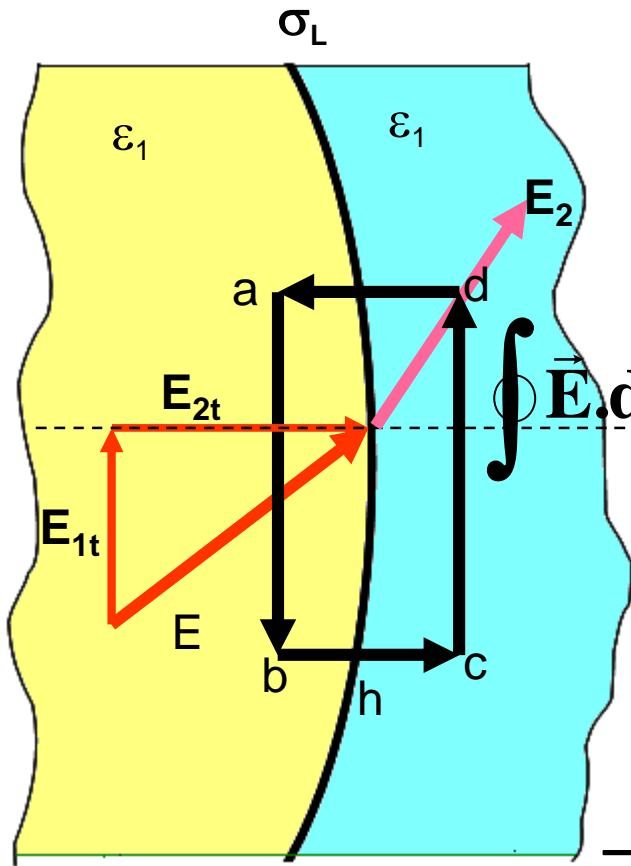
$$\sigma_L = 0$$

$$\mathbf{D}_{2n} - \mathbf{D}_{1n} = \mathbf{0} \Rightarrow \mathbf{D}_{2n} = \mathbf{D}_{1n}$$

$$\varepsilon_2 \mathbf{E}_{2n} - \varepsilon_1 \mathbf{E}_{1n} = \mathbf{0} \Rightarrow \mathbf{E}_{2n} = \frac{\varepsilon_1}{\varepsilon_2} \mathbf{E}_{1n}$$

$$\frac{\varepsilon_{r2}}{\chi_2} \mathbf{P}_{2n} - \frac{\varepsilon_{r1}}{\chi_1} \mathbf{P}_{1n} = \mathbf{0} \Rightarrow \mathbf{P}_{2n} = \frac{\varepsilon_{r1} \chi_2}{\varepsilon_{r2} \chi_1} \mathbf{P}_{1n}$$

Se conserva la componente normal \mathbf{D}



$$\oint \vec{E} \cdot d\vec{L} = 0$$

$$\oint \vec{E} \cdot d\vec{L} = \int_a^b \vec{E} \cdot d\vec{L} + \int_b^c \vec{E} \cdot d\vec{L} + \int_c^d \vec{E} \cdot d\vec{L} + \int_d^a \vec{E} \cdot d\vec{L} = 0$$

$$h \rightarrow 0$$

$$- \int_a^b E_{1t} dL + \int_c^d E_{2t} dL = 0$$

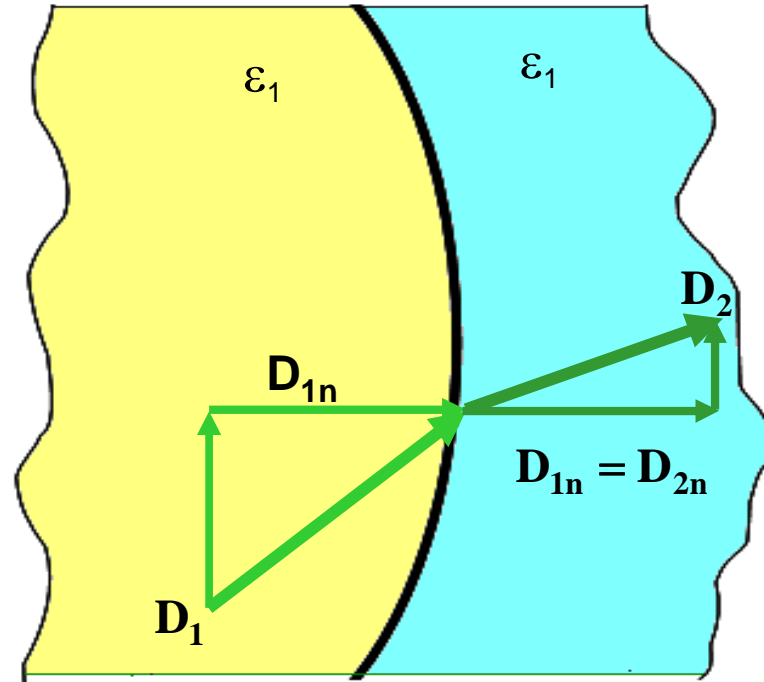
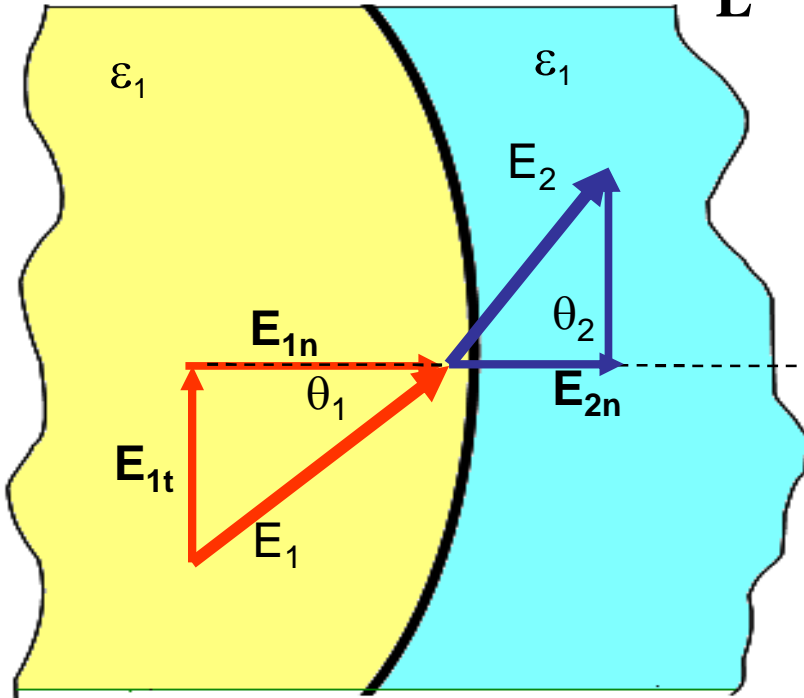
$$E_{1t} = E_{2t}$$

$$\frac{D_{1t}}{\epsilon_{r1}} = \frac{D_{2t}}{\epsilon_{r2}}$$

$$\frac{P_{1t}}{\chi_1} = \frac{P_{2t}}{\chi_2}$$

Se conserva la
componente tangencial de
E

$$\sigma_L = 0$$



$$\mathbf{E}_{2n} = \frac{\epsilon_1}{\epsilon_2} \mathbf{E}_{1n}$$

$$\mathbf{E}_{1t} = \mathbf{E}_{2t}$$

$$\mathbf{P}_{2n} = \frac{\epsilon_{r1} \chi_2}{\epsilon_{r2} \chi_1} \mathbf{P}_{1n}$$

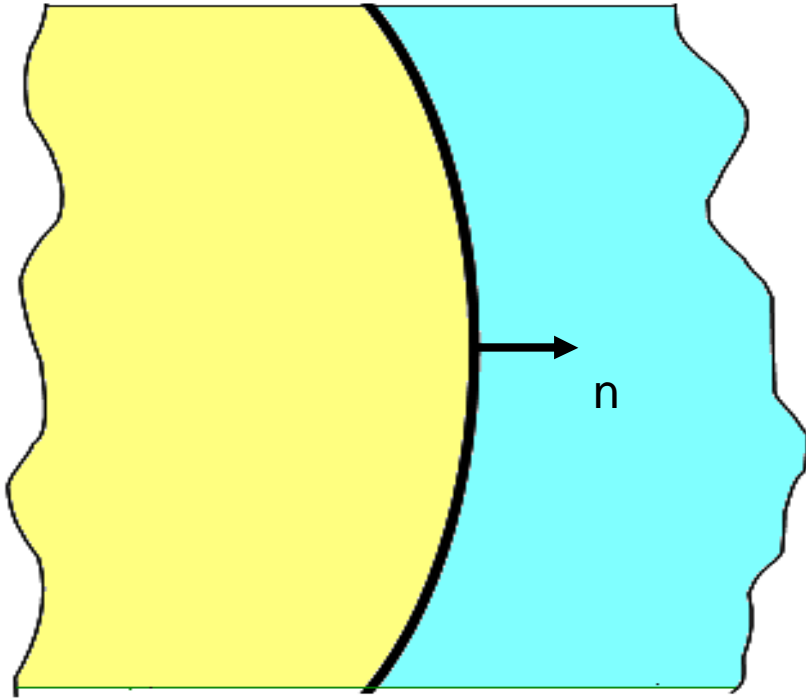
$$\mathbf{D}_{2n} = \mathbf{D}_{1n}$$

$$\mathbf{D}_{2t} = \mathbf{D}_{1t} \frac{\epsilon_{r2}}{\epsilon_{r1}}$$

$$\mathbf{P}_{2t} = \mathbf{P}_{1t} \frac{\chi_2}{\chi_1} =$$

D independiente del medio
Si \mathbf{D}_t es igual a \mathbf{e}_0

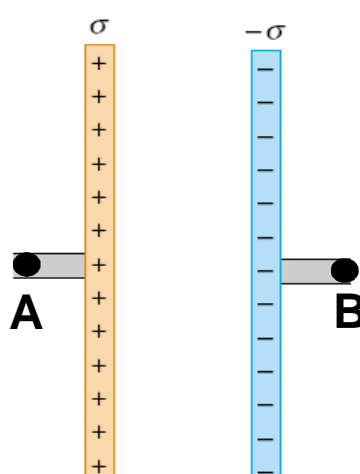
DENSIDAD DE CARGA SUPERFICIAL



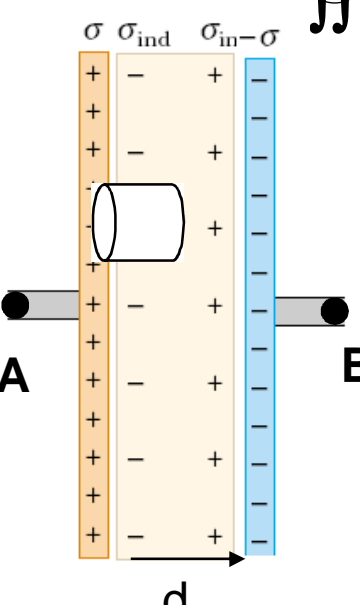
$$(\vec{D}_2 - \vec{D}_1) \cdot \hat{n} = \sigma_L$$

$$(\vec{P}_2 - \vec{P}_1) \cdot \hat{n} = -\sigma_{Pol}$$

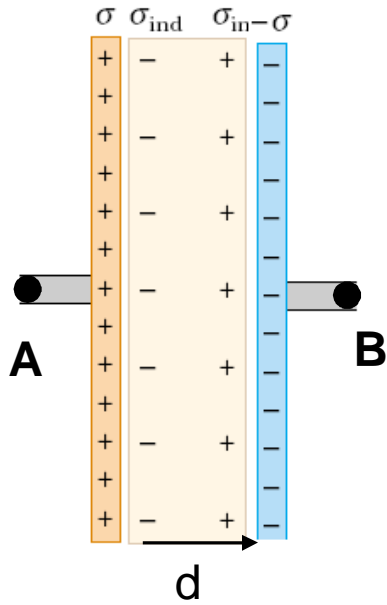
CAPACITOR DE PLACAS PLANO PARALELAS



$C_0 = \epsilon_0 \frac{A}{d}$
 $\vec{E} = \begin{cases} \frac{\sigma}{\epsilon_0} \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$
 $\vec{D} = \begin{cases} \sigma \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$
 $\Delta V_0 = \frac{\sigma}{\epsilon_0} d = \frac{Q}{\epsilon_0 A} d$



$\oiint \vec{D} \cdot d\vec{A} = q_L$
 $\vec{D} = \begin{cases} \sigma \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$
 $\vec{E} = \begin{cases} \frac{\sigma}{\epsilon_0 \epsilon_r} \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$
 $\Delta V = V_A - V_B = - \int_d^0 \vec{E} \cdot d\vec{l} = - \int_d^0 \frac{\sigma}{\epsilon_0 \epsilon_r} dx = \frac{\sigma}{\epsilon_0 \epsilon_r} d$
 $\Delta V > \Delta V_0$
 $C = \frac{Q}{\Delta V} = \frac{\sigma \cdot A}{\Delta V} = \epsilon_0 \epsilon_r \frac{A}{d}$
 $C > C_0$



$$\vec{D} = \begin{cases} \sigma \hat{x} & 0 < x < d \\ \mathbf{0} & \text{afuera} \end{cases}$$

$$\vec{E} = \begin{cases} \frac{\sigma}{\epsilon_0 \epsilon_r} \hat{x} & 0 < x < d \\ \mathbf{0} & \text{afuera} \end{cases}$$

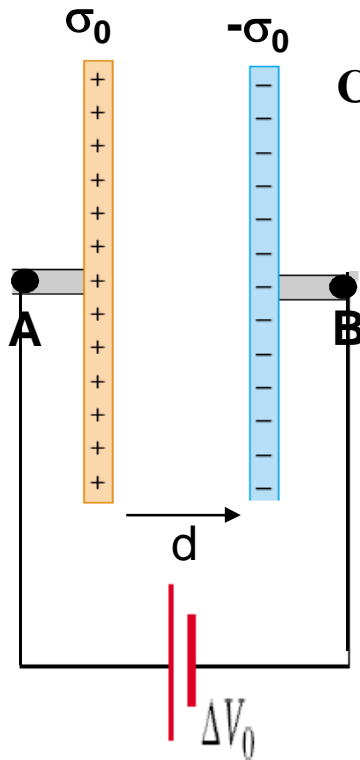
$$\vec{D} - \epsilon_0 \vec{E} = \vec{P}$$

$$\vec{P} = \begin{cases} \sigma \left(1 - \frac{1}{\epsilon_r}\right) \hat{x} & 0 < x < d \\ \mathbf{0} & \text{afuera} \end{cases} \quad (\vec{P}_2 - \vec{P}_1) \cdot \hat{n} = -\sigma_{\text{Pol}}$$

$$\sigma \left(1 - \frac{1}{\epsilon_r}\right) = -\sigma_{\text{Pol-}} \quad -\sigma \left(1 - \frac{1}{\epsilon_r}\right) = -\sigma_{\text{Pol+}}$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{C_0 \epsilon_r} = \frac{U_0}{\epsilon_r}$$

$$U < U_0$$



$$C_0 = \epsilon_0 \frac{A}{d}$$

$$\vec{E} = \begin{cases} \frac{\sigma_0}{\epsilon_0} \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$$

$$\vec{D} = \begin{cases} \sigma_0 \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$$

$$\Delta V_0 = \frac{\sigma_0}{\epsilon_0} d = \frac{Q_0}{\epsilon_0 A} d$$

$$\frac{\Delta V_0 \epsilon_0}{d} = \sigma_0 = \frac{Q_0}{A}$$

$$\oiint \vec{D} \cdot d\vec{A} = q_L$$

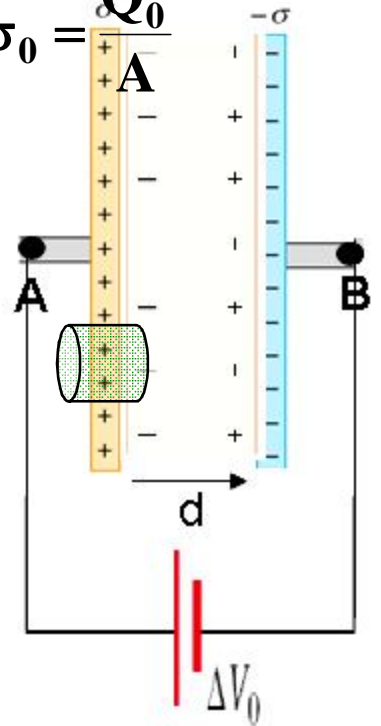
$$\vec{D} = \begin{cases} \sigma \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$$

$$\vec{E} = \begin{cases} \frac{\sigma}{\epsilon_0 \epsilon_r} \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$$

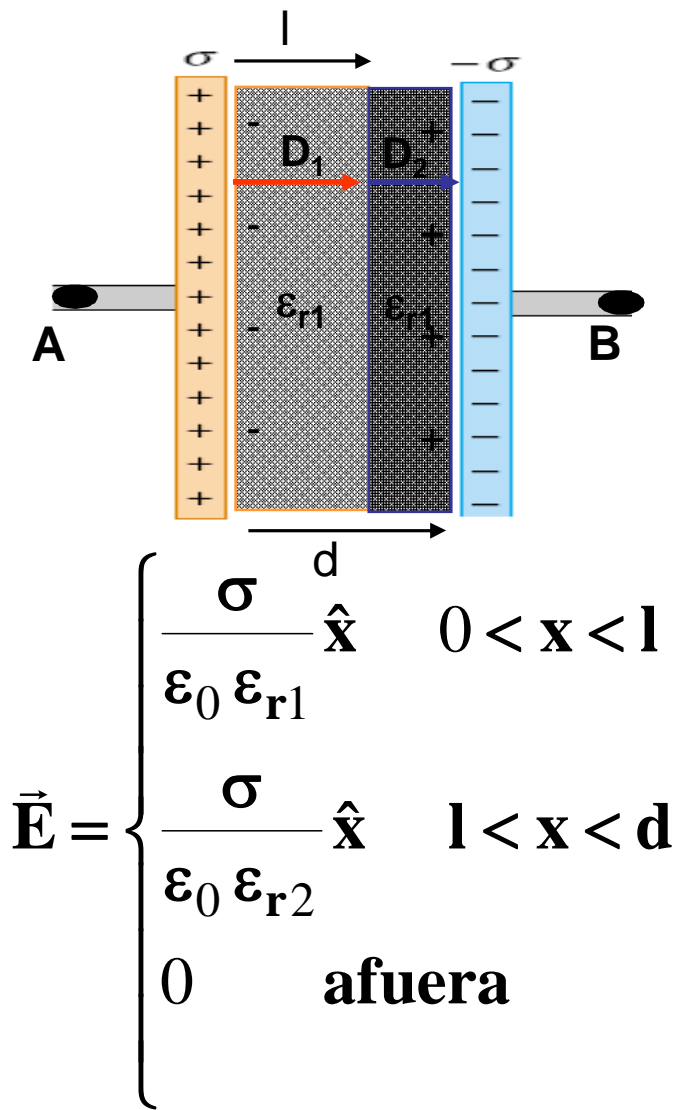
$$\left. \begin{aligned} \Delta V_0 &= \frac{\sigma}{\epsilon_0 \epsilon_r} d = \frac{Q}{\epsilon_0 \epsilon_r A} d \\ \frac{\Delta V_0 \epsilon_0 \epsilon_r}{d} &= \sigma_0 = \frac{Q_0}{A} \end{aligned} \right\}$$

$$\sigma > \sigma_0$$

$$U = \frac{1}{2} \Delta^2 V C = U_0 \epsilon_r$$



$$U_0 < U$$

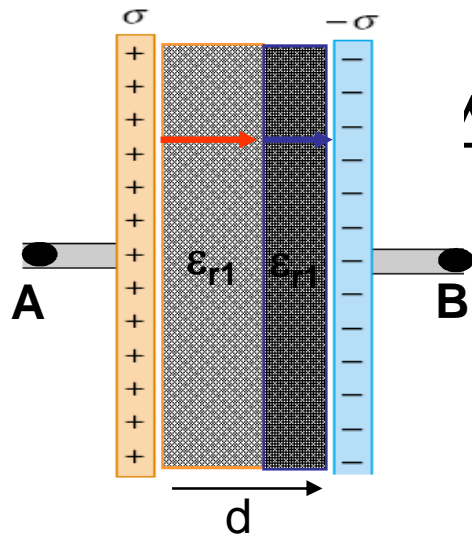


Sobre Sup. de separación $\sigma_L=0$

D es perpendicular a la superficie de separación
Entonces, D es independiente del medio $D_1=D_2$

$$\vec{D} = \begin{cases} \sigma \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$$

$$\vec{P} = \begin{cases} \sigma \left(1 - \frac{1}{\epsilon_{r1}} \right) \hat{x} & 0 < x < l \\ \sigma \left(1 - \frac{1}{\epsilon_{r2}} \right) \hat{x} & l < x < d \\ 0 & \text{afuera} \end{cases}$$

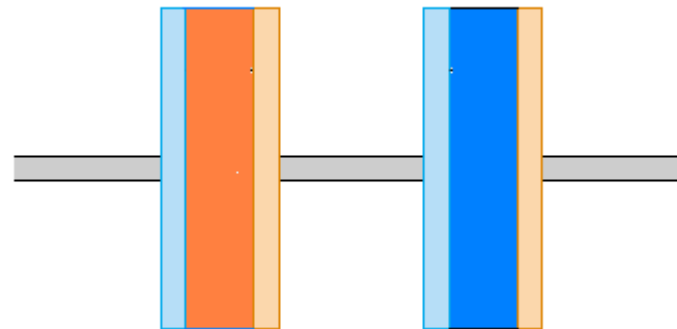


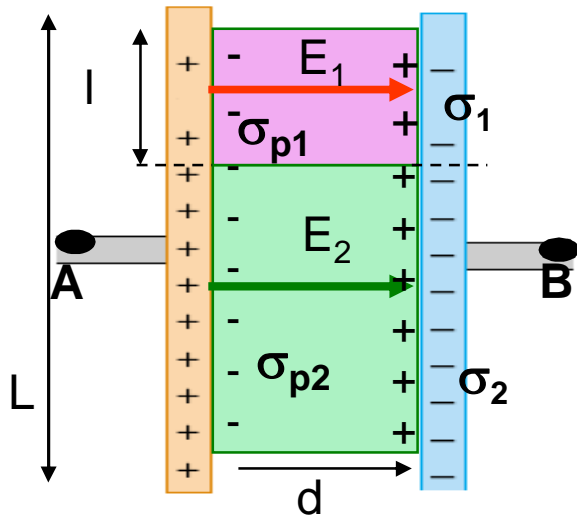
$$\Delta V = V_A - V_B = -\int_d^0 \vec{E} \cdot d\vec{l} = -\int_d^l \vec{E}_2 \cdot d\vec{l} - \int_l^0 \vec{E}_1 \cdot d\vec{l}$$

$$= -\int_d^l \frac{\sigma}{\epsilon_0 \epsilon_{r2}} dx - \int_l^0 \frac{\sigma}{\epsilon_0 \epsilon_{r1}} dx =$$

$$= \frac{Q}{\epsilon_0} \left(\frac{l}{\epsilon_0 \epsilon_{r1} A} + \frac{d-l}{\epsilon_0 \epsilon_{r2} A} \right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$





$$A_1 = l \cdot a$$

$$A_2 = (L - l) \cdot a$$

σ_L permanece constante . Sobre Sup. de separación $\sigma_L=0$,

E es paralelo a la superficie de separación

Entonces, E es independiente del medio $E_2 = E_1$

$$D_1 \neq D_2$$

$$Q = Q_1 + Q_2 = \sigma_1 A_1 + \sigma_2 A_2$$

$$E_1 = E_2 \Rightarrow \frac{\sigma_1}{\epsilon_0 \epsilon_{r1}} = \frac{\sigma_2}{\epsilon_0 \epsilon_{r2}}$$

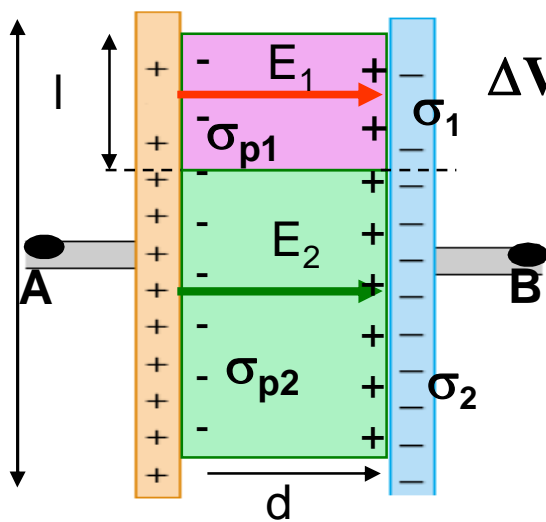
$$\sigma_2 = \frac{\sigma A - \sigma_1 A_1}{A_2}$$

$$\sigma_2 = \sigma_1 \frac{\epsilon_{r2}}{\epsilon_{r1}}$$

$$\sigma_1 = \sigma \frac{A}{A_1 + A_2 \frac{\epsilon_{r2}}{\epsilon_{r1}}} \quad \sigma_2 = \sigma \frac{A}{A_1 \frac{\epsilon_{r1}}{\epsilon_{r2}} + A_2}$$

$$\vec{E} = \begin{cases} \sigma \frac{1}{\epsilon_0} \left(\frac{A}{A_1 \epsilon_{r1} + A_2 \epsilon_{r2}} \right) \hat{x} & 0 < x < d \\ 0 & \text{afuera} \end{cases}$$

$$\vec{D} = \begin{cases} \sigma \frac{A}{A_1 + A_2 \frac{\epsilon_{r2}}{\epsilon_{r1}}} \hat{x} & 0 < x < d, 0 < y < L - l, \\ \sigma \frac{A}{A_1 \frac{\epsilon_{r1}}{\epsilon_{r2}} + A_2} & 0 < x < d, L - l < y < L, \\ 0 & \text{afuera} \end{cases}$$



$$A_1 = l \cdot a$$

$$A_2 = (L - l) \cdot a$$

$$\Delta V = V_A - V_B = - \int_d^0 \vec{E} \cdot d\vec{l} = - \int_d^0 \sigma \frac{1}{\epsilon_0} \left(\frac{A}{A_1 \epsilon_{r1} + A_2 \epsilon_{r2}} \right) dl$$

$$= \sigma \frac{d}{\epsilon_0} \left(\frac{A}{A_1 \epsilon_{r1} + A_2 \epsilon_{r2}} \right) = Q \left(\frac{d}{A_1 \epsilon_0 \epsilon_{r1}} + \frac{d}{A_2 \epsilon_0 \epsilon_{r2}} \right)$$

$$\frac{Q}{\Delta V} = C = \frac{d}{A_1 \epsilon_0 \epsilon_{r1}} + \frac{d}{A_2 \epsilon_0 \epsilon_{r2}} = C_1 + C_2$$

